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**PhD THESIS**

**ADVANCED METHODS OF SPECTRAL ANALYSIS USED FOR MODELING  
AND PREDICTING FINANCIAL TIME SERIES**

**== ABSTRACT ==**

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## SUMMARY

Introduction .....	6
1. Basics of probability theory, stochastic processes theory and signal theory.....	14
1.1 Basics of probability theory .....	14
1.1.1 Probability space .....	14
1.1.2 Random variables .....	15
1.1.3 Joint distribution of random variables .....	19
1.1.4 Random sequences in matrix notation .....	22
1.2 Basics of the theory of stochastic processes .....	26
1.2.1 The concept of stochastic process .....	26
1.2.2 Stationary processes.....	27
1.2.3 Convergence of random sequences .....	28
1.2.4 Continuity, derivability and integrability in square mean of random functions .....	29
1.3 Basics of signal theory .....	33
1.3.1 Types of signals .....	33
1.3.2 The noise type Signal .....	39
1.3.3 Some common signals .....	40
1.3.4 Time domain and frequency domain .....	46
1.3.5 Examples of frequency spectra .....	48
2. Classical spectral analysis .....	50
2.1. Periodic functions .....	50
2.2. Orthogonal functions .....	51
2.3. Systems of complete orthogonal functions .....	52
2.4. Fourier series .....	54
2.4.1 Definition.....	54
2.4.2 Bessel's inequality and Parseval's theorem .....	57

2.4.3	Generalized Fourier series .....	58
2.4.4	Operations on Fourier series .....	62
2.5.	Discrete Fourier Transform.....	67
2.5.1	Direct Discrete Fourier Transform .....	67
2.5.2	Inverse Discrete Fourier Transform .....	68
2.5.3	Pairs of Fourier Transforms .....	69
2.5.4	Fast Fourier Transformation (FFT) .....	70
2.6.	Nonparametric methods for estimating the spectrum .....	71
2.6.1	Using Periodogram in estimating the spectrum .....	72
2.6.2	Using FFT (DFT) in estimating the spectrum .....	80
2.6.3	Discrete Cosine Transform (DCT) .....	86
3.	Modelling and predicting exchange rates using Fourier spectral analysis .....	106
3.1	Application presentation and methods used .....	106
3.2	Description of computation procedures and results .....	107
4.	Advanced spectral analysis tTechniques: Independent Component Analysis and Singular Spectrum Analysis .....	125
4.1.	Independent Component Analysis .....	125
4.1.1.	Introduction .....	125
4.1.2.	Data preprocessing .....	127
4.1.3.	Approaches to extracting independent factors of a mixture of signals .....	128
4.1.4.	The problem of time series prediction using the ICA in combination with another prediction tool .....	132
4.2.	An advanced spectral method: Singular Spectrum Analysis .....	134
5.	Applications of independent component analysis and singular spectrum analysis to modeling and predicting financial time series .....	139
5.1.	Using independent component analysis to separating fundamental factors and smoothing of exchange series .....	139
5.2.	Modeling and predicting a 3-dimendionale time series of the price values (minimum, medium, maximum) of a financial asset, using Singular Spectrum Analysis .....	146

5.3. Using ICA in conjunction with SSA for robustifying the prediction of multidimensional time series .....	153
General conclusions .....	160
References .....	164



## **1. Introduction: literature review**

Spectral analysis has a long history. Its beginnings date back to the time of Pythagoras, who try to find musical harmony laws whose mathematical expression was found only in the eighteenth century, in terms of wave equation. Who first discovered a solution to the wave equation was Baron Jean Baptiste Joseph de Fourier in 1807 with the introduction of the Fourier series. Fourier's theory was extended to the case of arbitrary orthogonal functions by Sturm and Liouville in 1836. Sturm-Liouville theory led to the most successful empirical spectral analysis through the formulation of quantum mechanics by Heisenberg and Schrodinger in 1925 and 1926. In 1929 John von Neumann put the spectral theory of the atom on a firm mathematical foundation in his theorem of spectral representation in Hilbert space. Meanwhile, Wiener developed the mathematical theory of Brownian motion in 1923, and in 1930 introduced generalized harmonic analysis, ie, spectral representation of a stationary random process. The joint support of von Neumann and Wiener spectral representations is Hilbert space. In 1942 Wiener applied his methods to the problems of prediction and filtering, and his work has been interpreted and expanded by Norman Levinson. Wiener, in his empirical work focuses more on autocorrelation function than the power spectrum. Modern history begins with the discovery of spectral estimation by J.W. Tukey in 1949, which is the statistical equivalent of Fourier's discovery. However, spectral analysis has been costly in terms of computation. It became available with the publication in 1965 of the fast Fourier transform algorithm of JS Cooley and JW Tukey. Cooley-Tukey's method has allowed practical waveforms signal processing in both the time domain and frequency domain, something that was not possible with the continuous systems. Fourier Transform became not only a theoretical description, but also a tool. With the development of FFT the spectral analysis empirical field has grown in importance and is now a major discipline. Further contributions were: the introduction of maximum entropy spectral analysis of

John Burg in 1967; the development of spectral windows by Emmanuel Parzen and others since the 1950s; the statistical work of Maurice Priestley and the hypothesis testing in time series analysis by Peter Whittle, since 1951; the Box-Jenkins approach of George Box and GM Jenkins in 1970; the autoregressive spectral estimation and criteria for determining the order introduced by E. Parzen and H. Akaike since 1960; and so on.

More recently, the classical methods of spectral analysis have added a number of new methods, considered part of the field of computational intelligence, such as Singular Spectrum Analysis (SSA), Independent Component Analysis (ICA) and Wavelets analysis (WA). The first two methods (SSA and ICA) will be the subject of research in this thesis. We will focus mainly on their application potential in the analysis, nonparametric modeling, noise filtering (smoothing) and prediction of financial series.

Singular Spectrum Analysis (SSA) belongs to the class of signal processing methods based on subspaces. An important development was the formulation of the spectral decomposition of the covariance operator of stochastic processes by Kari Karhunen and Michel Loève in the late 1940s (Loeve, 1945, Karhunen, 1947).

Broomhead and King (1986) and Fraedrich (1986) proposed the use of SSA and multi-channel SSA (M-SSA), in the context of non-linear dynamics, in order to reconstruct the attractor of a time series measured system. These authors have offered an extension and a more solid idea of reconstructing the dynamics of a single time series, based on the embedding theorem. Other authors have applied simple versions of the M-SSA to weather and environmental datasets (Colebrook, 1978 Barnett and Hasselmann, 1979 Weare Nasstrom, 1982).

Ghil, Vautard and colleagues (1989, 1991, 1992, 2002) observed the analogy between the trajectory matrix of Broomhead and King, on the one hand, and the Karhunen-Loeve decomposition (Principal Component Analysis in time), on the other hand . Thus, SSA can be used as a method in the time domain and frequency domain

for time series analysis - independent of attractor reconstruction and even in cases where the latter may fail.

The so-called "Caterpillar" methodology is a version of SSA, which was developed independently in the former Soviet Union. This methodology has become known in the world recently (Danilov and Zhigljavsky 1997, Zhigljavsky 2010, Golyandina et al. 2001, Golyandina and Zhigljavsky 2013). "Caterpillar-SSA" emphasizes the concept of separability, a concept that leads, for example, to specific recommendations regarding the choice of SSA parameters.

In the last two decades, the Blind Source Separation (BSS) by Independent Component Analysis (ICA) received special attention due to its potential applications in signal processing, such as systems for speech recognition, telecommunications and medical signal processing. The purpose of ICA is to recover independent sources with only observations from sensors that are linear unknown mixtures of independent unobservable source signals. Unlike transformations based on correlation, such as Principal Component Analysis (PCA), ICA not only decorrelates signals (up to 2nd order statistics), but also reduces higher order statistical dependencies, trying to make signals as independent as possible.

There were two different areas of research that were considered in Independent Component Analysis. On the one hand, the study of separating mixed sources observed in a number of sensors was a classic and difficult signal processing problem. The pioneering work on blind source separation is due to Jutten, Herault and Guerin (1988). They proposed an adaptive algorithm in an architecturally simple feedback. The rule learning was based on a neuro-mimetic approach and was able to separate simultaneous unknown independent sources. This approach has been explained and developed by Jutten and Herault (1991), Comon (1991), Karhunen and Joutsensalo (1993), Cichocki and Moczczynski (1992) and others. Moreover, Comon (1994) introduced the concept of analyzing the independent component and proposed functions of cost related to minimizing mutual information between sensors.

On the other hand, in parallel with studies on blind source separation, unsupervised learning rules based on information theory were proposed by Linsker

(1992), Becker and Hinton (1992) and others. The idea was to maximize the mutual information between the inputs and outputs of a neural network. This approach is related to the reduction of redundancy that has been suggested by Barlow (1961) as a strategy for coding neurons. Each neuron would codify features that are statistically independent compared to other neurons.

Bell and Sejnowski (1995) were the first to explain the problem of blind source separation in terms of information theory and their application to separation and deconvolution of sources. Their adaptive methods are more plausible from a neuronal processing perspective than cost functions based on the cumulants proposed by Comon. A similar adaptive method, but "non-neuronal", of source separation was proposed by Cardoso and Laheld (1996).

Other algorithms have been proposed from different perspectives: the approach based on estimating maximum likelihood was first proposed by Gaeta and Lacoume (1990), the approach based on Negentropy maximization of Girolami and Fyfe (1996), the nonlinear PCA algorithm developed by Karhunen and Joutsensalo (1994) and Oja (1995). Lee, Girolami and Sejnowski (1997) provides a unifying framework for the problem of source separation by explaining the relationship between algorithms different from each other. The learning rule derived is optimized when using natural gradient (Amari, 1997) or relative gradient (Cardoso and Laheld, 1996).

The algorithm originally proposed by Bell and Sejnowski (1995) was adapted to separate the super-Gaussian sources. To overcome this limitation were developed other techniques that were able to separate simultaneously sub- and super-Gaussian sources. Pearlmutter and Parra (1996) obtained an ICA learning rule generalized from the maximum likelihood estimate where it models explicitly the source base distribution that was supposed to be fixed in the original algorithm. While estimating density was cumbersome and required a sufficient amount of data, the algorithm was able to separate a large number of sources with a wide range of distributions. An algorithm simple and highly effective was proposed by Girolami and Fyfe (1996) through an approach based on maximizing negentropy. Lee, Girolami and Sejnowski (1997) derived the same learning rule of Infomax approach that preserves the simple architecture and shows superior speed of convergence.

For usual applications, the instant mixing model may be appropriate because the propagation delays are negligible. However, in real environments may appear substantial delays and this requires an architecture and an algorithm to account for mixing sources delayed in time and sources in convolution. The problem of multichannel blind sources separation was approached by Yellin and Weinstein (1994) and Ngyuen and Jutten (1995) and others, using criteria based on the 4th order cumulants.

ICA and SSA are advanced spectral analysis methods and fairly new. They are generally applicable to the signal processing, providing openings to a variety of potential applications. In this thesis are explored applications of ICA and SSA to analysis, modeling, noise filtering (smoothing) and prediction of financial time series.

## **2. The structure of the thesis**

The structure of the thesis includes an introduction, 5 chapters and conclusions. Three of the five chapters (1,2 and 4) are theoretical and the other two (3 and 5) are applicative.

The first chapter, called "**Basics of probability theory, stochastic processes theory and signal theory**" introduces the mathematical background of the classical theory of spectral analysis. Underlying all these theories are clearly defined notions and concepts of probability theory.

The objects of interest in spectrum estimation are the stochastic (random) processes. They represent the fluctuations in time of a certain quantity that can not be fully described by deterministic functions. The dynamics of economic and financial variables, such as foreign exchange rates, or stock index daily fluctuations, are examples of random processes. Formally, a random process is defined as a collection of random variables indexed with respect to time. The index set is infinite and can be continuous or discrete. If the index set is continuous, the random process is known as a

continuous-time random process, and if the index set is discrete, it is known as a discrete-time random process.

In this thesis we focus only on processes in discrete time where the index set is the set of integers.

The signal theory is also essential for spectral analysis, particularly in relation to issues in discrete time signal sampling and their quantization in amplitude. Signal analysis provides two types of approaches: the time domain and in the frequency domain.

The second chapter called "**Classical spectral analysis**" provides a broad overview of classical spectral theories that have as a starting point the Fourier series and the Fourier transform. The primary objective in spectrum estimation is determining the Power Spectral Density (PSD) of a random process. PSD is a function that plays a fundamental role in stationary random process analysis where it quantifies the total power distribution in frequency. The PSD estimation is based on a set of observed process data samples. A necessary assumption is that the random process is stationary at least in a broad sense, i.e., its statistics of first and second order does not change over time. The PSD estimation provides information about the structure of a random process that can then be used for modeling, predicting or filtering the observed process. Of particular interest is given in this Chapter to Discrete Fourier Transform (direct and inverse) and Fast Fourier Transform (direct and inverse). We carefully discuss the main nonparametric estimation methods of spectrum, among which Periodogram and Fast Fourier Transform play an important role.

The third chapter, entitled "**Modelling and predicting exchange rates by using Fourier spectral analysis**" suggests an application of classic spectral analysis to modeling financial series. By their nature, these are usually nonstationary, while Fourier techniques address stationary series in a broad sense. Their use implies a prior stationarization of series. The application is based on four procedures: a parametric adjustment procedure to eliminate its trend for stationarizing the series; the second procedure uses Fast Fourier Transform (FFT) that makes the transition from the

representation of the signal in the time domain (amplitude vs. time) to the representation in the frequency domain (amplitude versus frequency); the third procedure uses the Inverse Fast Fourier Transform (IFFT) by which the FFT results are reconverted back in time, not before to filter frequencies for smoothing the representation (to reduce noise); the fourth case involves an extrapolation of the IFFT curve by an equation that is often used to calculate the Discrete Fourier Transform, which allows us to predict the time series on a reasonable prediction horizon. The results are encouraging regarding the predictive capability of the method, given that the problem of prediction in itself is particularly difficult, hovering at the limit of unpredictability.

The 4th chapter entitled "**Advanced techniques of spectral analysis: Independent Component Analysis and Singular Spectrum Analysis**" presents two methods of advanced spectral analysis (ICA and SSA), on which we have already referred in introduction. Specific algorithmic aspects of the two methods are discussed.

The 5th chapter, entitled "**Application of independent component analysis and singular spectrum analysis to modeling and predicting financial time series**" is applicative in nature and explores the potential of the two methods in the financial area that is not part of the usual field of application (engineering). We present three applications. The first (**Using Independent Component Analysis for separating fundamental factors and smoothing exchange time series**) uses exclusively ICA (i.e., one computationally efficient variant of this, FastICA). The data set consists of 6 parallel series of exchange rates, which is considered to represent observable mixtures of a set of latent (unobservable) fundamental factors, but are supposed to represent the data generating mechanism for the observable mixtures. The role of ICA is to reveal these factors, i.e., to carry out "blind separation of sources". Separation of such fundamental causal factors is very important for multivariate analysis of financial time series in order to explain past co-developments and to anticipate future developments. In addition, ICA allows smoothing time series by eliminating the spectral components of low amplitude. The second application (**Modeling and prediction of a 3-**

**dimensionale time series representing prices (minimum, medium, maximum) of a financial asset, using Singular Spectrum Analysis)** uses an extension of ICA to multidimensional time series. The dataset used in this application consists of a 3-dimensional time series of asset prices (minimum, medium, maximum) for the company Antibiotice SA. The method performs a nonparametric modeling of the series, its smoothing, and finally its prediction on a certain time horizon. For prediction two methods are used: a recursive one and a vector one. SSA is part of Computational Intelligence methods and is known for its accurate predictions, even when series are nonstationary and strongly nonlinear. The third application (**Using ICA in conjunction with SSA for the robustification of multidimensional time series prediction**) uses a hybrid approach that combines the two methods, ICA-SSA. Empirical studies have suggested that the mixtures we actually observe may be better predicted indirectly, through latent underlying factors (independent components) revealed by ICA, followed by remixing the predictions made by independent components to produce predictions for observed mixtures. In this sense, ICA can be regarded as a technique for the robustification of multidimensional time series (random vectors) prediction. The prediction in the space of independent components (disclosed applying FastICA) is done with SSA. The rationale for preferring the prediction of independent components is that they usually offer more structured and regular representations, so accurate predictions. The predictions are then remixed to produce forecasts of observable mixtures. The dataset used in this application contains time series of asset prices for 5 companies listed on the Bucharest Stock Exchange (BSE). Experiments confirm the usefulness of ICA-SSA combined approach to improve predictions.

The last part of this thesis is dedicated to the overall conclusions and main contributions of the author.

Economic and financial modeling is traditionally focused on approaches and methods in the time domain (real domain); there are only rare situations where approaches and methods in the frequency domain (complex domain) are preferred. The main contribution of this thesis is that of proposing nonconventional modeling

techniques and strategies for the financial application field. In addition, it extends the interest of the classical methods of spectral analysis, towards advanced techniques in the category of computational intelligence, such as SSA and ICA. The research is based on an extensive literature review. Applications were implemented in Matlab and passed all stages of experimental testing and validation.

### **3. Applications. Own contributions**

#### **3.1. Modelling and predicting exchange rates using Fourier spectral analysis**

Predicting currency markets or stock markets can be very difficult. In an attempt to achieve this, we will use various mathematical algorithms, such as parameter estimation of the process trend (with the purpose of eliminating it and hence stationarising the series), followed by applying spectral analysis: Fourier Transform, Inverse Fourier Transform and prediction.

I written a program in Matlab, which attempts to predict exchange rates for the near future.

First the for 458 days are loaded, and then are divided into two distinct datasets; the first data set consists of the first 365 observations and the second data set will contain observations from 366 to 458 (the next 3 months).

The most interesting aspect in this application is the extrapolation of FFT curve. The extrapolation of FFT curve depends on how many points we can choose in the process of cleaning the spectrum of amplitudes that results from FFT. Keeping in analysis only dominant frequencies and eliminating frequencies with minor amplitudes (reflecting rather circumstantial factors, associated with the idea of noise) plays an extremely important role in the improvement of predictions for future trends (increase or decrease in series values).

Choosing a long prediction horizon in the case of currency markets or stock markets is hazardous and is not a good option. Indeed, if we could accurately predict the currency market or stock market every time we make decisions about the type or volume of currency to be purchased from it allows obtaining profits. Unfortunately, this is not possible due to the fact that predictability of markets is weak. These variables depend on numerous internal factors and external conditions that give them a high variability, enhanced by all sorts of unforeseeable events that may have appear in economy or society. Therefore, choosing a prediction horizon of 90 days

wassomewhat excessive and should be avoided in reality . Even a prediction for the next week or the next day is critical in this field of applications. The fact that predictions on a long horizon were reasonable enough, shows that spectral analysis is still capable to capture certain important features related to the structure and internal dynamics of financial processes, despite their difficulty to be predicted.

### **3.2. Using independent component analysis to separate fundamental factors and smoothing exchange series**

In this application we use Independent Component Analysis (ICA) for revealing fundamental, latent (unobservable) factors, but supposed to represent the generating mechanism of observation data we considered: 6 parallel series exchange rates. ICA belongs to a wider research area, called Blind Source Separation (BSS). Each measured signal is regarded as a mixture of several distinct factors underlying it. Separation of such fundamental causal factors is very important for multivariate analysis of financial time series in order to explain co-developments of the past and to anticipate future developments.

The proposed approach is based on the interpretation of financial time series as mixtures of several distinct fundamental factors. In an attempt to produce more structured and regular representations and more accurate predictions, we employ an approach based on spectral decomposition using ICA, for separating, smoothing and remixing fundamental factors. This consists in exploiting the ability of ICA to disclose independent components behind several series of parallel exchange rates.

The prediction problem is addressed in the following two applications. The first one uses Singular Spectrum Analysis for prediction. The last application considers a prediction strategy that combines the two advanced spectral analysis methods presented in Chapter 4: ICA and SSA.

### **3.3. Modeling and prediction of a 3-dimendionale time series representing prices (minimum, medium, maximum) of a financial asset, using Singular Spectrum Analysis**

An advanced spectral method called **Singular Spectrum Analysis (SSA)** is used in this application to modeling and predicting a 3-dimensionale time series. SSA is a nonparametric technique and is directed to detect the structure of time series. It combines the advantages and complements other methods, such as Fourier analysis and regression analysis. The dataset used in this application consists of a 3-dimensional time series values (minimum, medium, maximum) of asset prices for the company Antibiotice SA, the leading producer of generic drugs in Romania, which was listed on the Bucharest Stock Exchange in April 1997.

We use a sample spanning a period of 2674 days of trading.

Results from the extension of multidimensional Singular Spectrum Analysis, a powerful nonparametric method for time series analysis, proved to be very promising. MSSA was used for modeling, smoothing and predicting a 3-dimensional time series representing asset prices (minimum-maximum-average) recorded for Antibiotice SA, the leading producer of generic drugs in Romania. Experimental evidence demonstrates the ability of the method to perform well in case of multidimensional time series and to produce accurate forecasts for a relatively long prediction horizon.

### **3.4. Using ICA in conjunction with SSA for robustifying the prediction of multidimensional time series**

The advantage of ICA is the ability to decompose a mixture consisting of the set of random variables in statistically independent components. Empirical studies have suggested that mixtures that we observe may actually be better predicted indirectly, through their underlying latent factors (independent components) revealed by ICA, followed by remixing predictions made on independent components to produce predictions for observed mixtures. In this sense, ICA can be regarded as a technique for robustification of multidimensional time series prediction (random vectors).

The dataset used in this application contains time series of asset prices for 5 companies listed on the Bucharest Stock Exchange (BVB), as follows: CNTEE Transelectrica SA, SNTGN Transgaz SA, SN Nuclearelectrica SA, SNGN Romgaz SA, Antibiotice SA.

The prediction strategy used was a combined method in which ICA is used in combination with another prediction method. The aim of ICA is to predict financial time series observed as mixtures of signals by first performing predictions in the ICA space (generated by independent components), and then making the reconstruction of predictions for the initial time series by returning to the space of observable mixtures. Predictions can be done separately and with a different method for each component independently, according to its temporal structure.

Financial time series can be regarded as mixtures of several distinct fundamental factors. This application has addressed the blind separation of sources, to reveal the underlying factors behind the 5 parallel asset prices in an attempt to produce a more structured and regular representations and more accurate predictions. Experiments confirm the usefulness of ICA-SSA combined approach to multidimensional modeling of time series. This approach has greater adaptability to suit the different characteristics of various time series data.

## **GENERAL CONCLUSIONS**

Time series models used routinely in modeling economic and financial processes are models in the time domain. Thereby misses a fundamental dimension characterizing implicitly financial series in particular: the frequency domain. Financial series are high- and very high-frequency series, because the transactions on currency exchange and stock markets are at a fast pace, facilitated by the electronic environment that hosts them. Series representing transactions "intra-day" have the ability to generate a huge volume of data in a very short period. In addition, variability in the data is very high as well as the frequency of these variations.

Time series analysis in the frequency domain is common to many areas of natural sciences and technical sciences, where it has a considerable tradition.

This thesis aims to recover this type of analysis in the frequency domain for economic and financial science and practice. With this theme we believe that it is trying to cover a deep interest.

In the first part of the thesis I analyzed the classical spectral analysis techniques, whose foundations were laid long time ago by Jean Baptiste Joseph Fourier. This area has continuously progressed and continues to be effervescent today.

I found it necessary to introduce the fundamentals of classical spectral analysis by introducing the main nonparametric methods for estimating the spectrum (parametric methods were not subject of this thesis). The most important among these nonparametric methods are the periodogram, the modified periodogram and the Fast Fourier Transform (direct and inverse).

In chapter 3 we made a wide illustration of how these classical spectral estimation techniques may be used for smoothing and predicting financial series.

In addition, in this thesis we have been concerned with studying and practical use of advanced spectral analysis techniques. More specifically, we analyzed and used two such techniques: Singular Spectrum Analysis and Independent Component Analysis. Using these techniques in modeling economic and financial processes is unique because these areas are not a traditional application field. The reason of choosing these techniques derives from their great potential in modeling processes with features that are not convenient for traditional methods: lack of stationarity, nonlinearity, a large amount of disturbance in the data. Unlike conventional methods, both methods (SSA as well as ICA) have the capacity to work with such time series in terms of providing accurate predictions. Note that financial series are estimated to be at the limit of unpredictability, being associated with random walk. In these conditions, predictions on a short or medium term are not considered credible and the short term predictions are themselves difficult to face with due to uncertainty in financial markets.

Emerged as a powerful tool for modeling and prediction in meteorology, geophysics and related disciplines in natural sciences, singular spectrum analysis may

still be applied in many different areas. Financial and economic time series presents some characteristics that make them suitable for an analysis based on SSA in smoothing, trend extraction and trend forecast. Both theoretical developments and practical results strongly support the use of SSA in financial time series analysis. As a method of non-parametric modeling, smoothing, extracting trend and cyclical components, and forecast, respectively, we believe that SSA must be part of the practitioners' toolkit, it can give results that are similar or better than those obtained by the traditional methods. As a method for forecasting the economic and financial time series, SSA seems to work very promising. A number of possible applications remain open and can be pursued in future research.

On the other hand, independent component analysis (ICA) is a statistical technique that aims to reveal the hidden factors that underlie sets of random variables, measurements, or signals. ICA defines a generative model for multidimensional observed data, which is usually given as a large database. In the model, variables are assumed to be linear or nonlinear mixtures of some unknown latent variables and the mixing system is also unknown. Latent variables are supposed to be Non-Gaussian and are mutually independent, and they are called independent components of observed data. These independent components, called sources or factors can be found by ICA. We believe that the observed financial series enroll well into this pattern. For example, parallel series expressing exchange rates, or parallel series expressing indexes of the various financial markets tend to present co-developments that are caused by a number of fundamental, latent (unobservable) factors, but which are common sources of influence for the full set of observed parallel series. We so therefore consider these parallel series as observable mixture generated by hidden fundamental factors. A method to determine those factors that dictate the evolution of fundamental variables in many markets can be of real interest. ICA is one such tool and its use in economics and finance is thus fully justified.

ICA can be seen as an extension of principal component analysis and factor analysis. It is however a more powerful technique, capable of finding such factors or latent sources and succeeds where these classical methods fail completely. ICA is not only useful to decorrelate of a set of statistical series, but also to seek for independent

factors. Or we know that independence is a property tighter than the noncorrelation. In this fact lies the strength of ICA when used for breaking down the components of time series.

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