Experimental determinations of the eigenmodes for composite bars made with carbon and Kevlar-carbon fibers

C M Miriţoiu¹, M M Stănescu², C O Burada³, D Bolcu³ and V Roşca³
¹Faculty of Mechanics, University of Craiova, 165 Calea Bucuresti, 200620, Craiova, Romania
²Department of Applied Mathematics, University of Craiova, 13 Al Cuza, 200396, Craiova, Romania
³Faculty of Mechanics, University of Craiova, 165 Calea Bucuresti, 200620, Craiova, Romania

E-mail: miritoiucosmin@yahoo.com

Abstract. For modal identification, the single-point excitation method has been widely used in modal tests and it consists in applying a force in a given point and recording the vibratory structure response in all interest points, including the excitation point. There will be presented the experimental recordings for the studied bars (with Kevlar-carbon or carbon fibers), the frequency response function in Cartesian and polar coordinates. By using the frequency response functions we determine the eigenparameters for each bar. We present the final panel of the eigenmodes (with the damping factors, eigenfrequencies and critical damping) for each considered bar. Using the eigenfrequency of the first determined eigenmode, the bars stiffness has been determined. The presented bars can be used in practical engineering for: car or bus body parts, planes body parts, bullet-proof vests, reinforcements for sandwich beams, and so on.

1. Introduction

Many papers present various studies regarding Kevlar or carbon-fibers studies. In this sense, in [1] there is presented a study regarding the interfacial bonding under bending and uniaxial compression for sandwich structures made by carbon-fiber and aluminum honeycomb with and without Kevlar-fiber interfacial toughening. In [2] there is presented a research on impacted sandwich composites with Kevlar/hybrid and carbon face sheets subjected to different temperatures. Testing was performed to determine bending and core shear stresses, maximum energy absorption, and absorbing energy, moment parameter, performance parameter and compression strength after impact. Other studies regarding these fibers can also be found, but not limited, in [3, 8].

The modal identification is characterized by theoretical and experimental procedures used to determine the system eigenmodes parameters. The procedures used for tests can be divided in two important big groups: single-point excitation and multiple-point excitation. In this paper, starting from the theoretical background of modal identification, it is established an experimental method used to determine the eigenmodes for some composite bars with Kevlar-carbon and carbon fibers with epoxy resin. The epoxy resin is RESOLTECH 1050 type and its hardener is RESOLTECH 1058 type (having a mixed density of 1.11 g/cm³, mixed viscosity of 633 MPa·s at 23°C, 5% elongation to break and 6% flexion to break). There was used the single-point excitation method to determine the bars
eigenmodes. This method has been widely used in modal testing and it consists in applying a force in a given point and recording the vibratory response in all interest points, including the excitation point, if needed. Although the single-point excitation requires a minimum of equipment, it needs a laborious analysis to perform extensive result processing in order to interpret the dynamic behavior of the structure under test.

2. The modal parameters identification procedure

The theoretical procedure used in this paper has been previously presented by the authors in papers like [10] and [11]. So, according to [9], [10] and [11], if a mechanical system is described by \( n \) concentrated mass points jointed with elastic elements (with the next characteristics: \( s_k \) stiffness and \( \mu_k \) damping) and can have \( n \) degrees of freedom, and if it is loaded with an \( \{\Theta(t)\} \) external excitation, the motion equations can be given by equation (1).

\[
[G] \cdot \{v(t)\} + [M] \cdot \{\dot{v}(t)\} + [S] \cdot \{\ddot{v}(t)\} = \{\Theta(t)\}.
\]  

(1)

In the equation (1) we have marked with: \([G]\), \([M]\) and \([S]\) the matrices of mass, damping and stiffness; \(\{v(t)\}\) with its first and second derivatives are the displacement, velocity and acceleration vectors; \(\{\Theta(t)\}\) is the generalized forces vector.

The equation (2) is used to determine the system response at external excitation. Its response is processed as a sum of \( n \) modal contributions due to each separate degree of freedom (as described in equation (2)).

\[
\sum_{k=1}^{N} \left( \frac{1}{a_k \left( \mu_k - i(\omega - \nu_k) \right)} \cdot \{\Theta(\omega)\} \right) + \sum_{k=1}^{N} \left( \frac{1}{a_k \left( \mu_k - i(\omega + \nu_k) \right)} \cdot \{\Theta(\omega)\} \right) = \{v(\omega)\}
\]  

(2)

In the equation (2) we have marked with: \(\{v(\omega)\}\) is the displacement Fourier transform; \(\{\varphi^k\}\) and \(\{\bar{\varphi}^k\}\) is the eigenvector and its complex conjugate \( k \) order; \(\mu_k\) is the damping factor \( k \) order; \(\nu_k\) is the damped natural frequency \( k \) order; \(a_k\) and \(\bar{a}_k\) are the norm constants of eigenvector; \(\omega\) is the external excitation frequency.

In the experimental modal identification, the modal vectors can be replaced by two constants. Their value can be determined with equation (3).

\[
2 \cdot V^k_{ij} = \frac{\varphi^k_i \bar{\varphi}^k_j}{a_k} + \frac{\varphi^k_i \varphi^k_j}{a_k}, \quad 2 \cdot i U^k_{ij} = \frac{\varphi^k_i \bar{\varphi}^k_j}{a_k} - \frac{\varphi^k_i \varphi^k_j}{a_k},
\]  

(3)

According to the modal identification literature, we may insert the system admittance parameter as the ratio between the displacement response and the force excitation. This parameter may be calculated by using the equation (4).
The concept of discrete system with concentrated mass in \( n \) material points was used in the approximations adopted during the mathematical model \([9, 10, 11]\). In order to obtain an accurate approximation of the real system by the discrete one, \( n \) must converge to infinity. Because of experimental and processing technique and of the necessary time for data processing, this is impossible. The frequencies domain is limited to a reasonable width in practical applications, which is obtained by the major resonances of the analysed equipment and the frequency domain of the application goal. In these conditions, the sum from equation (4) is reduced to several components marked in the following with \( n \), too. The contribution of superior and inferior modes are included in two corrections factors known as residual flexibility \( S'_{ij} \) (for superior modes) and inferior modal admittance \(-\left(M'_{ij} \cdot \omega^2\right)\) (for inferior modes) \((\text{according to [9, 10, 11]})\). Using these notations, the equation 4 will have the form like in equation (5).

\[
\sum_{k=1}^{n} \frac{U^k_{ij} + iV^k_{ij}}{\mu_k + i(-\omega + \nu_k)} - \sum_{k=1}^{n} \frac{U^k_{ij} + iV^k_{ij}}{\mu_k + i(-\omega - \nu_k)} + S'_{ij} - \left(M'_{ij} \cdot \omega^2\right)^{-1} = \Psi_{ij}(\omega),
\]

(4)

The modal identification of a system with \( n \) degrees of freedom assumes determination of \( 4n \) modal parameters: \( \mu_k, \nu_k, U_{ij}, V_{ij} \). These are intrinsic characteristics of the system, independent of the external conditions.

3. The modal identification for the chosen composite bars

In figure 1 there is presented the experimental setup and the analyzed composite bars (a built-in end and the other one is free with an accelerometer placed at the middle of the free length). The bars are marked with 1, 2 and 3 according to figure 1. The samples have the thickness of 0.002 m, the width 0.05 m and the density: 1090 kg/m \((\text{sample 1})\), 1170 kg/m \((\text{sample 2})\) and 1260 kg/m \((\text{sample 3})\).

![Figure 1](image)

**Figure 1.** Experimental setup: a) the studied samples; b) a scheme with the experimental setup.
In order to identify the eigenmodes, the next steps were followed:
- we have marked with 1 and 2 the samples with Kevlar-carbon fibers (the difference between them is the direction of fiber application, as shown in figure 1) and with 3 the sample with carbon fiber;
- the experimental recordings are determined (the impact force made with an impact hammer with the sensitivity 1,020 pC/ms$^{-2}$ and the vibratory response with an accelerometer with 0,04 pC/ms$^{-2}$ sensitivity);
- the frequency response functions (abbreviated as FRF in the next paragraphs) are determined in both Cartesian and Nyquist coordinates;
- the resonances approximate position and the initial modal parameters $\mu_k$ and $\nu_k$ calculus (where $k$ is the number of the modal parameter);
- the first stage identification of modal parameters $\mu_k$, $\nu_k$, $U_{ij}$, $V_{ij}$, $S'_{ij}$ and $-(M'_{ij})^{-1}$ on limited frequency domains (the identification is made by using linear procedures, determining those modal parameters that, inserted in relation (5), generate theoretical characteristics which approximate with minimal error the experimentally determined frequency response function);
- the final stage of modal parameters $\mu_k$, $\nu_k$, $U_{ij}$, $V_{ij}$, $S'_{ij}$ and $-(M'_{ij})^{-1}$ over the entire domain (the identification is made by using nonlinear procedures of recursive approximation, determining those modal parameters that, inserted in equation (5), generate theoretical characteristics which approximate with minimal error the experimentally determined frequency response function).

The tests are performed in order to determine the bars eigenmodes, eigenfrequencies, the damping factors and the critical damping for each eigenmode.

**Figure 2.** Sample 1: a) Experimental recordings; b) FRF in Cartesian coordinates.

**Figure 3.** Sample 1: a) Real and imaginary parts of FRF; b) FRF in polar coordinates.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Eigenmode</th>
<th>$\mu$ (miu)</th>
<th>$\nu$ (niu)</th>
<th>U</th>
<th>V</th>
<th>$\zeta$ (zita)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>[-(Ns/m)/kg]</td>
<td>[1/s]</td>
<td></td>
<td></td>
<td>[(Ns/m)/kg]</td>
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Using the relation (6) we have determined the loss factor for each eigenmode.

$$\eta = 2 \cdot \zeta$$

We have obtained the next values:
- sample 1: 0.038 (first eigenmode); 0.018 (second eigenmode); 0.016 (third eigenmode);
- sample 2: 0.040 (first eigenmode); 0.014 (second eigenmode); 0.012 (third eigenmode);
- sample 3: 0.032 (first eigenmode); 0.018 (second eigenmode); 0.012 (third eigenmode).

Using relation (7), from [13], we have determined the bars flexural rigidity.

$$\nu \approx 0.1591549 \cdot \frac{(EI)^{0.5} \cdot 4.694}{(\rho A)^{0.5} \cdot l^2}$$
4. Conclusions
In this paper, the modal parameters were determined for three types of composite bars marked in this way: sample 1 and 2 – bars with Kevlar-carbon fibers with different orientation and sample 3 – a bar with carbon fiber. All the fibers were combined with epoxy resin. The single-point excitation method was used. For all the bars, from the experimental setup, three eigenmodes were determined. The above presented experiment shows many distinct eigenmodes in a large frequency domain: from 94.378 s\(^{-1}\) up to 2138.874 s\(^{-1}\). The modal identification method has the advantage that it is non-destructive and can be used in the case of complex structures built from modern (like composite materials, shape memory alloys, and so on) or classical (like steel, aluminum or cast iron structures) materials. But this method also presents some disadvantages because it can be used for some mechanical parameters calculus (for example, the ones that characterize the breakage of a certain structure). We have determined the loss factor using the critical damping value (\(\zeta (zita)\)) and the bars stiffness. In practical engineering, these types of composites structures can be used for: car or bus body parts, planes body parts, bullet-proof vests, reinforcements for sandwich beams, and so on.

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