Phase Coordinate System and p-q Theory Based Methods in Active Filtering Implementation

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Abstract—This paper is oriented towards implementation of the main theories of powers in the compensating current generation stage of a three-phase three-wire shunt active power system. The system control is achieved through a dSPACE 1103 platform which is programmed under the Matlab/Simulink environment. Four calculation blocks included in a specifically designed Simulink library are successively implemented in the experimental setup. The first two approaches, namely those based on the Fryze-Buchholz-Depenbrock theory and the generalized instantaneous reactive power theory, make use of phase quantities without any transformation of the coordinate system and provide the basis for calculating the compensating current when total compensation is desired. The others are based on the p-q theory concepts and require the direct and reverse transformation to/from the two-phases stationary reference frame. They are used for total compensation and partial compensation of the current harmonic distortion. The experimental results, in terms of active filtering performances, validate the control strategies implementation and provide arguments in choosing the most appropriate method.

Index Terms—active filters, harmonic distortion, power conditioning, power system control, real time systems.

I. INTRODUCTION

Due to the inevitable and increasing use of nonlinear loads in both industrial and domestic electrical systems, the power quality issue is now, more than ever, a great concern for utilities and their customers.

Compared with other solutions for power conditioning, such as specially connected transformers, series reactors and passive filters, the active power filtering is a versatile and flexible solution which allows the total compensation, in terms of harmonic distortion cancellation, power factor correction and load unbalance compensation.

When connected in parallel with the load, the so called shunt active power filter (SAPF) injects a controlled compensating current in the point of common coupling (PCC) and needs to be sized only for the compensating apparent power.

The power switching devices in the structure of the common voltage source inverter (VSI) based SAPF are Pulse Width Modulation (PWM) controlled in accordance with the desired compensation strategy. Thus, after generating the compensating current based on the sensed supply voltage and load current, the closed-loop control of the compensation system must be able to handle the accurate tracking of this reference current.

There are many theories that have been implemented over time in the three-phase SAPFs’ control [1-20]. The first category is where no coordinate system transformation is required in order to generate the reference compensating current. It mainly includes the Fryze-Buchholz-Depenbrock (FBD) theory [1-3], the generalized instantaneous non-active power (GINAP) theory [4-7] and the generalized instantaneous reactive power (GIRP) theory [8-12]. A three phase notch filter is proposed in [13] to eliminate the fundamental components of the load current and generate the compensating currents.

In contrast, the control based on the so called p-q theory of the instantaneous reactive power in three-phase circuits involves the transformation of the voltage and current phase coordinate system into the orthogonal stationary reference frame followed by the reference compensating current identification, in order to generate the desired compensating currents, is based on the p-q theory.

An interesting approach, which leads to optimized values of the power factor and current total harmonic distortion, consists in the control of the supply current shape through the admittances of each harmonic component [21-22].

Equally, to guarantee the accurate and fast tracking of the reference currents, attention is directed to the control loops design and digital control implementation [18], [21-27].

In this paper, section II introduces the implemented structure of the three-phase three-wire SAPF. Next, the implementation of the reference compensating current calculation methods on the dSPACE platform is presented. Section IV is devoted to the experimental setup end results. Conclusions are drawn at the end of the paper.

II. STRUCTURE OF THE ACTIVE FILTERING SYSTEM

In the adopted three-phase three-wire structure, the SAPF consists of a full bridge VSI with a capacitor on the DC side, which is connected to the PCC through a coupling inductor in order to attenuate the high switching frequencies at the VSI’s output (Fig. 1).

After measuring the load currents and SAPF’s output currents on two phases together with the supply voltages and DC-bus voltage, the conditioned sensor signals of the acquisition system supply the analog-to-digital converters (ADC) of the control system which is based on dSPACE 1103 board for rapid prototyping.

The control algorithm of the compensating currents and DC-bus voltage is implemented in the Matlab/Simulink environment, and then executed on the dSPACE platform.

Digital Object Identifier 10.4316/AECE.2013.01012

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The reference compensating currents ($i_{FA}^*$, $i_{FB}^*$, and $i_{FC}^*$) are calculated in accordance with the desired compensation strategy, whereas the additional reference currents ($i_{FA}'$, $i_{FB}'$, and $i_{FC}'$) are provided by the DC-voltage controller in order to maintain the voltage on the DC-side by covering the losses in the power circuit. A Proportional Integral (PI) controller was designed to handle the DC-bus voltage [23]. The quick compensation current controllability and the robustness under parameters variation is ensured by using a hysteresis-band controller. The maximal value of the switching frequency is limited by the proper choice of the coupling inductor and by the hysteresis band value.

III. COMPENSATING CURRENT CALCULATION

A Matlab/Simulink library has been created and developed as part of the experimental platform to generate the desired compensating current based on different time-domain approaches [19].

Irrespective of the implemented algorithm, the reference compensating current calculation (RCCC) block is fed by the three supply voltages and load currents and, after the specific processing stage, delivers the three compensating currents to the control circuit. The associated line currents drawn from the power supply after compensation ($i_{sA}^*$, $i_{sB}^*$, and $i_{sC}^*$) are calculated too (Fig. 2). Depending on the calculation algorithm, either the desired supply currents or the reference compensating currents are generated first and then, the others are expressed based on following relationship:

\[ i_{sk}^* = i_{sk}^* + i_{Lk}, \quad k = A, B, C. \]  

When the compensation goal is the removal of both current harmonic distortion and reactive power, i.e. the total compensation, the remaining line currents are the active components of the load currents ($i_{Lk}$, $k=A,B,C$), which are the minimal currents ensuring the same active power as the total currents under imposed voltage conditions.

Thus, the reference compensating currents can be simply expressed as:

\[ i_{sk}^* = i_{Lk} - i_{Lk}, \quad k = A, B, C \]  

A. Phase Coordinate System Based Methods

The most resounding theory of powers for multi-phase circuits in the time domain is the FBD theory published by Depenbrock in German in 1962 [1] and in English in 1993 [2].

The FBD-based implementation in Fig. 3 follows the fundamentals of the method for three-phase circuits. Thus, the collective active power ($P_k$) and the square of the collective rms value of the voltages ($U_k^2$) are calculated, in order to calculate the equivalent conductance ($G$) as their ratio.

Then, the phase active currents are generated as:

\[ i_{Lk} = i_{sk}^* = G \cdot u_k, \quad k = A, B, C \]  

Figure 1. The configuration of the three-phase three-wire shunt active power filtering system

Figure 2. The Simulink block for reference compensating current calculation

Figure 3. The content of block RCCC based on FBD theory implementation
Third-order Butterworth filters with the passband edge frequency of 100π rad/s were used to calculate the average values of the instantaneous quantities.

While the calculation of the active current through FBD theory creates prerequisites to obtain unity power factor (UPF) after compensation, the implementation of the generalized instantaneous non-active power theory introduced by Peng and Tolbert in 2000 [4] is more flexible as it allows obtaining either UPF or perfect harmonic cancelation (PHC) through compensation. In concrete terms, the definition of the active current vector,

\[ i_p(t) = \frac{P(t)}{U_p(t)} u_p(t), \]

makes use of a reference voltage vector \( u_p(t) \) which can be associated either to the supply voltages (for UPF strategy) or to the fundamental components of the supply voltages (for PHC strategy). Moreover, the averaging interval in the active power \( P(t) \) and the rms voltage \( U_p(t) \) calculation can be limited to an integer multiple of one-half fundamental period in order to compensate periodic currents [5].

Another generalized theory for three-phase power systems is the so-called generalized instantaneous reactive power theory proposed by Peng and Dai in 1996 [8]. The application of this theory for the reactive power and harmonic compensation was then presented in [9].

In accordance with GIRP’s theory, the instantaneous current vector is the sum of instantaneous active current vector \( i_p \) and instantaneous reactive current vector \( i_q \), i.e.,

\[ i = i_p + i_q = \frac{p}{u} u + \frac{q \times u}{u \cdot u}, \]

where

- \( u \) is the instantaneous voltage space vector;
- \( \cdot \) and \( \times \) denote the dot and cross products of two vectors;
- \( p = u \cdot i = u_a i_a + u_b i_b + u_c i_c \);
- \( q = u \times i = \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix} = \begin{bmatrix} u_b c_i - u_c i_b \\ u_c a_i - u_a i_c \\ u_a b_i - u_b i_a \end{bmatrix} \).

Following directions of the currents in Fig. 1, expression (1) can be used to express the reference compensating vector of a SAPF as follows:

\[ \mathbf{i}_{pF}^* = \left( \mathbf{i}_{pF}^* + \mathbf{i}_{qF}^* \right) = \left( \frac{p^*}{u} u + \frac{q^* \times u}{u \cdot u} \right). \]

The Simulink block in Fig. 4 leads to the compensation of both harmonic and reactive power, which means that:

\[ p_F = p - P; \quad q_F = q, \]

where \( P \) is the average value of the load instantaneous active power \( p \) over the fundamental period.

B. \( p-q \) Theory Based Methods

The \( p-q \) theory of the instantaneous powers was formulated by Akagi, Kanazawa and Nabae in 1983 [14] in terms of instantaneous active and reactive powers \( (p \text{ and } q) \) calculated by using the components of voltage and current space vectors \( (u \text{ and } i) \) in the stationary reference frame \( (u_a, u_b, i_a, i_b) \). Based on the relationship between the current, voltage and apparent power vectors and taking into consideration the Clarke’s power invariant transformation from A-B-C frame to \( \alpha-\beta \) frame, the current vector can be expressed as [17]:

\[ i = \frac{2}{3} \frac{u}{|u|} (p - jq), \]

where \( |u| \) is the modulus of the voltage space vector.

After decomposing the two instantaneous powers into their DC \( (P \text{ and } Q) \) and AC \( (p \text{ and } q) \) components, different components of the current vector can be highlighted [17]. They can be used to express either the reference compensating current vector or the reference supply current vector, depending on the desired compensation strategy and its implementation.

To reduce the amount of calculation, when the total compensation is the final objective, the desired supply current vector, which is the active current, is calculated first, as shown in Fig. 5.

\[ i_s^* = i_{sa}^* + j i_{sb}^* = \frac{2}{3} \frac{u}{|u|} \left[ p - j (q - Q) \right]. \]

The associated Simulink schema is shown in Fig. 6.
IV. EXPERIMENTAL IMPLEMENTATION AND RESULTS

All the blocks of the Matlab/Simulink library discussed in the previous section have been included in an experimental three-phase three-wire shunt active filtering platform.

The versatile DS1103 PPC Controller Board was programmed from the Matlab/Simulink environment through the Real-Time Interface and the Real-Time Workshop was used to generate C code for the real-time application.

As shown in Fig. 7, the sensed and normalized quantities from analog to digital converters are processed in accordance with the control algorithm and the IGBT’s gating signals are sent through digital output channels.

The imposed sampling time was of 17 μs for all the implemented control strategies, which is the minimal value that could be obtained to execute the generated C-codes in real time.

The VSI consists of a Semikron SKM100GB123D-type IGBT three-legs bridge. The main parameters of the active filtering system are shown in Table I.

The nonlinear load whose effects must be compensated is a three-phase fully-controlled thyristor-bridge rectifier with a resistive–inductive load. As known, the total harmonic distortion factor (THD) of the current (about 27%) is mainly due to harmonics pairs of orders 5 and 7, 11 and 13, 17 and 19 (Fig. 8).

The imposed DC-voltage value of 740 V is handled by the control circuit. Thus, just before starting the compensation, the DC-voltage set value is ensured by a little increasing of the supply current which is drawn by the VSI (Fig. 9).

The dSPACE ControlDesk environment was used as a means of managing and instrumenting the real-time experiments.

The ControlDesk panel in Fig. 10 shows the SAPF behavior when the total compensation task is accomplished by implementing the FBD theory. As expected, the generated compensating current is accurately tracked by the actual compensated current, so that the supply current is nearly sinusoidal and its fundamental component has the same phase as the supply voltage. It can be seen that the low order harmonics are practically missing in the harmonic spectrum of the supply current after compensation (Fig. 11).

Similarly, the UPF compensation strategy has been successfully implemented by using the GIRP theory and then the p-q theory based blocks, as illustrated through the waveforms in Fig. 12.

The partial compensation results in terms of current distortion removing (Fig. 13) has been achieved through the appropriate block of reference compensating current generation according to the p-q theory approach.

<table>
<thead>
<tr>
<th>TABLE I. THE MAIN PARAMETERS OF THE SAPF SYSTEM</th>
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<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>SAPF rated power (kVA)</td>
</tr>
<tr>
<td>Line-to-line supply voltage (V rms)</td>
</tr>
<tr>
<td>Supply fundamental frequency (Hz)</td>
</tr>
<tr>
<td>DC-capacitance (µF)</td>
</tr>
<tr>
<td>Coupling inductance (mH)</td>
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<tr>
<td>Current hysteresis band (A)</td>
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</tbody>
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Figure 6. The content of block RCCC based on p-q theory implementation for partial compensation.

Figure 7. The compiled Simulink schema of the control system.

Figure 8. The load current and its harmonic spectrum.

Figure 9. The DC-voltage (a), the supply current (b) and the current drawn by inverter (c) just before starting the compensation.
TABLE II. SUMMARY OF THE COMPENSATION PERFORMANCES

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$I_{es}$ (A)</th>
<th>$P_{es}$ (kW)</th>
<th>$S_{es}$ (kVA)</th>
<th>$PD_{es}$ (%)</th>
<th>$PF_{es}$</th>
<th>$DPF_{es}$</th>
<th>$I_{ls}$ (A)</th>
<th>$P_{ls}$ (kW)</th>
<th>$S_{ls}$ (kVA)</th>
<th>$PD_{ls}$ (%)</th>
<th>$PF_{ls}$</th>
<th>$DPF_{ls}$</th>
<th>$P_{es}/P_{ls}$</th>
<th>$I_{es}/I_{ls}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FBD</td>
<td>14.93</td>
<td>8.046</td>
<td>9.777</td>
<td>27.47</td>
<td>0.823</td>
<td>0.858</td>
<td>12.92</td>
<td>8.424</td>
<td>8.46</td>
<td>4.17</td>
<td>0.995</td>
<td>0.999</td>
<td>6.588</td>
<td>1.047</td>
</tr>
<tr>
<td>GIRP</td>
<td>14.83</td>
<td>8.136</td>
<td>9.573</td>
<td>26.88</td>
<td>0.849</td>
<td>0.883</td>
<td>13.29</td>
<td>8.541</td>
<td>8.586</td>
<td>4.38</td>
<td>0.995</td>
<td>0.999</td>
<td>6.137</td>
<td>1.050</td>
</tr>
<tr>
<td>$p$-$q$</td>
<td>15.00</td>
<td>8.157</td>
<td>9.792</td>
<td>27.29</td>
<td>0.833</td>
<td>0.868</td>
<td>13.29</td>
<td>8.577</td>
<td>8.622</td>
<td>4.33</td>
<td>0.995</td>
<td>0.999</td>
<td>6.303</td>
<td>1.051</td>
</tr>
<tr>
<td>$p$-$q$ partial</td>
<td>14.90</td>
<td>7.128</td>
<td>9.774</td>
<td>28.38</td>
<td>0.729</td>
<td>0.761</td>
<td>15.13</td>
<td>7.440</td>
<td>9.921</td>
<td>4.31</td>
<td>0.75</td>
<td>0.759</td>
<td>6.585</td>
<td>1.044</td>
</tr>
</tbody>
</table>

The supply current waveforms in Fig. 10 and Fig. 12 show that the total compensation performances are similar. The comparative quantitative analysis (Table II) allows highlighting the following aspects:
- the global power factor after compensation ($PF_S$) has the same value of 0.995 for the three methods;
- the displacement power factor ($DPF_S$) is practically equal to 1;
- the active filtering efficiency, expressed as the ratio of the highest value (4.38 %) corresponds to the GIRP strategy;
- the active filtering efficiency, expressed as the ratio of the supply current harmonic distortion factors before and after compensation.
filtering, is over 6; the highest value (6.588) is obtained through the FBD strategy and the lowest value (6.137) is obtained through the GIRP strategy;
- the ratio of the load active power ($P_L$) and the supply active power after compensation ($P'_S$) illustrates the weight of losses in the active filter circuit, which is of about 5% of the load power; the rms value of the current after filtering is diminished by about (10÷14)%.

The partial compensation results (Fig. 13 and Table II) show the following particularities:
- the PHD value decreases from 28.38% to 4.31%, i.e. the filtering efficiency is of about 6.585;
- as the distortion component of the load current is less than the active component of the current through SAPF (which ensure the coverage of losses and a constant value of the DC voltage), there is a slight increase of about 1.5% in the global rms value of the supply current; of course, the supply current after filtering lags the voltage and the values of supply and load DPF are approximately the same.

V. CONCLUSION

The paper presents a synthesis of all aspects related to the implementation of four SAPF control strategies for total and partial compensation based on FBD, GIRP, and p-q theories. A specific dSPACE 1103 platform that allows working under the Matlab/Simulink environment has been used to implement the control system. The calculation relationships of the analyzed strategies have been included in a Simulink library as four blocks with the same inputs/outputs.

As the minimal sampling time was the same (17 µs), even the calculation time is different from one strategy to another, it follows that the communication time for signals acquiring and signals sending has a decisive weight. Under these conditions, the active filtering performances are close. In all cases, the harmonic distortion factor is below 5%. It reveals that, the filtering performance improvement can be obtained not so much by the adopted method of reference current generation as by the hardware system.

However, when total compensation is desired, the lower computing requirements in implementing the FBD theory should be reflected in better filtering performance by using a faster hardware support On the other hand, the p-q theory makes possible to meet more objectives of compensation.

REFERENCES