

## Preface

*The Universe is made up of stories,  
not of atoms.  
Muriel Rukeyser*

The main purpose of the present book is to support the ideas that just like continuity, whose proper structures are topologies, discreteness has its own structures. When involving usual metrics, the topologies essentially refine the property of sub-additivity, known as *rule of a triangle*: each side is less than the sum of the others. The fact is that most people strongly renounce this rule and its consequence “*The shortest route between two points is a straight line*”. However, the reversed inequality is realistic and present in a lot of aspects of our existence. The book highlights several cases of super-additive phenomena, with the aim to increase interest in finding unifier structures of discreteness.

Most frequently, when we are looking for super-additivity, we arrive at chapters of hyperbolic mathematics; this remark justifies the title. The word *Complements* in the title suggests that the book brings only a few additional facts to the vast field of hyperbolic mathematics, particularly, in connection with super-additivity and related topics. Such complements are still useful since traditionally, most works on fields naturally carrying super-additivity (e.g. indefinite inner products, etc.), avoid discussion on this topic.

The book has a didactical character. From the huge volume of existing literature on the discussed topics, it extracts the most representative results in the study of the super-additivity. The incipient aspect of the problematic and the simplicity of the text make the book accessible to young students, starting at the high school level. It does not aim to be facile popular book, but a rigorous one, which could offer a solid base for further studies. Of course, the experienced may directly enter Chapter V, where *horistologies* play the role of structures of discreteness.

The initial educational curriculum usually regards the knowledge of the material, palpable and visible aspects of the reality. This is a natural order of things, but a time may come when we ask about immaterial and invisible entities of the same reality, like events, causality, evolution, etc. Then, we have to turn, *become like children*, and look for a book like this.

The book contains a lot of unexpected properties of a universe of events. Thus, there exists no triangle in the classical sense of an *angle*, but often, the Euclidean properties have peers in terms of events (see Chronogeometry in Section IV.1); two events in spatial relation have neither a closest cause, nor a closest effect (see Multi-lattices in Section IV.2); in the incomplete universes of events (like our ?!) there are not e-emergent

sequences of events because these universes lack necessary e-germs, hence we cannot find the essential cause of such sequences (see Section V.4); etc.

From a meta-theoretical point of view, the present book is a combination of several physical and mathematical theories, which contain fundamental arguments in favor of structural discreteness. The greatest physical theories that operate with discreteness are Relativity and Quantum Physics: The twin's paradox means super-additivity of proper time, while quanta and principles of uncertainty are features of discreteness. In mathematics, the theory of Indefinite Inner Product Spaces is most interfacing with discreteness, via the super-additivity of the intrinsic norms. Among these theories, Einsteinian Relativity has the greatest involvement in the text, with examples, physical interpretations and even with terminology. The fact that the universe of relativist events, presented in real variables, is a natural example of indefinite inner product space, assures the integrated study of these two theories.

Einstein himself remarked the discrete character of the universe of events. In a letter to Walter Dallenbach (1916), he wrote: *The problem seems to me how one can formulate statements about a discontinuum without calling upon continuum space-time as an aid; the latter should be banned from the theory as a supplementary construction not justified by the essence of the problem, which corresponds to nothing "real". But we still lack the mathematical structure unfortunately.*

The remaining question is to what degree do horistologies satisfy Einstein's request. Of course, the answer shall come latter, from specialists, in so far as they consider that it deserves the effort.

\* \* \*

Undoubtedly, I could never have realize this book without the support of the *Alexander von Humboldt Stiftung*. During 1974, under Professor Werner Heisenberg's presidency, this foundation granted a Stipendium to me, with the plan to search for the proper structures of super-additivity and discreteness. Therefore, I address my sincere gratitude to all those who gave that opportunity to me.

I acknowledge all my professors who have guided me on this strange and unfriendly road *from super-additivity to discreteness*. I am also indebted to the scientific reviewers for their kindness to read the manuscript and give a lot of valuable suggestions.

Last but not least, I am grateful to all my colleagues who were interested in my work and have brought their own contributions to horistology.

*The author,  
Craiova, November 2016*