How can the IRP p-q theory be applied for active filtering under nonsinusoidal voltage operation?

Abstract. This paper presents an analysis of Akagi’s p-q theory for active filtering under nonsinusoidal voltage conditions. It is ascertained that compensating the alternative parts of the instantaneous active and reactive powers leads to a sinusoidal power supply current only under sinusoidal voltage conditions. A modified definition of the active component of the current is proposed for operation under nonsinusoidal voltage conditions and shunt active filter applications. A generic example and the full controlled rectifier-based DC motor drive system are used as case studies.

Streszczenie. Artykuł przedstawia analizę teorii p-q Akagi’ego, stosowanej do filtracji aktywnej w warunkach napięć niesinusoidalnych. Stwierdzono, że kompensacja składników oscylacyjnych mocy chwilowych, czynnej i bieżej, prowadzi do sinusoidalnego prądu zasilania tylko w warunkach zasilania napięciem sinusoidalnym. Zaproponowano zmodyfikowaną definicję składowej czynnej prądu na potrzeby sterowania równoległego filtru aktywnego pracującego przy niesinusoidalnym napięciu zasilania. Przykładowy napęd silnika stałego został użyty w artykule jako przykład dla przedstawionej w artykule analizy. (Jak teoria IRP p-q może być użyta do filtracji aktywnej w warunkach niesinusoidalnego napięcia?)

Keywords: p-q theory; active current; nonsinusoidal voltage; active filtering.

Stwórka kluczowe: teoria p-q, prąd aktywny, napięcie niesinusoidalne, filtracja aktywna.

Introduction

Making evident the components of the current, especially the active one, is an old concern of researchers in straight connection with the need for finding efficient methods for improving power factor. Recent efforts have been concentrated on the development and control of active power filters. Thus, when total compensation is expected, the active filter has to provide the current vector

\[ i_F = i_L - i_D, \]

where \( i_L \) and \( i_D \) are the load current vector and its active component.

The development of active filtering techniques has rendered topical the theory of instantaneous complex power. This theory was first introduced as a unitary concept, by V. Nedelcu [1, 2] and it was developed by other authors who used it for grounding certain active filtering techniques [3]. Thus, the p-q theory of the instantaneous reactive power developed by Akagi and his coauthors [4] provides the mathematical foundations for the active filters control and it is about to become a means of identification and analysis of powers under nonsinusoidal current and/or voltage operation [5, 6, 7]. Many extensions of the original p-q theory have been developed, including the four-wire systems case. On the other hand, the p-q theory developed by Akagi has a lot of deficiencies, especially from physical point of view. Some conceptual limitations of this theory were pointed out by Willems in [8, 9].

Moreover, Professor L.S. Czarnecki from Louisiana State University has investigated how power phenomena and properties of three-phase systems are described and interpreted by the p-q theory [10, 11]. The argumentation through which Czarnecki disagrees with the p-q theory is principally based on the relativity of the active and reactive character of the currents defined by Akagi and his followers [12]. The Currents’ Physical Components introduced by Czarnecki took as a starting point the Fryze’s, Shepherd’s and Zakikhani’s powers theories in single-phase circuits under nonsinusoidal conditions [13]. The attention was directed on both power phenomena and general methods for improving the power factor. At present, this theory is applied to single and three-phase circuits with unbalanced and harmonic generating loads. It gives the interpretation of physical phenomena that affect the powers under nonsinusoidal and unbalanced conditions and can provide fundamentals for power compensators design.

Correct interpretation of p-q theory

In order to obviate any ambiguity in current components interpretation, a possible decomposition of the current space-vector takes into account the DC components (\( P \) and \( Q \)) and the AC components (\( p_\cdot \) and \( q_\cdot \)) of the instantaneous powers \( p \) and \( q \) which are revealed in the complex apparent power \( s \) expression, i.e.

\[ s = \frac{3}{2} u \cdot i^* = p + jq = P + p_\cdot + j(Q + q_\cdot). \]

Thus, the expression of the current vector becomes

\[ i = \frac{2}{3} \frac{1}{|u|^2} u \left[ (P + p_\cdot) - j(Q + q_\cdot) \right]. \]

Starting from this expression, the active current vector \( i_a \), the reactive current vector \( i_r \) and the supplementary useless current vector \( i_d \) can be defined as follows:

\[ i_a = \frac{2}{3} \frac{P}{|u|^2} \left( u_d + ju_q \right), \]

\[ i_r = \frac{2}{3} \frac{Q}{|u|^2} \left( u_q - ju_d \right), \]

\[ i_d = \frac{2}{3} \frac{1}{|u|^2} \left( u_d p_\cdot + u_q q_\cdot + ju_q p_\cdot - u_d q_\cdot \right). \]

As regards the sum of \( i_e, i_d \) and \( i_d \) moduli, we found that

\[ |i_e|^2 + |i_d|^2 + |i_d|^2 = \frac{4}{9} \frac{1}{|u|^2} \left( P^2 + Q^2 + p_\cdot^2 + q_\cdot^2 \right) = \frac{8}{9} \frac{Pp_\cdot + Qq_\cdot}{|u|^2}. \]

By integrating (7), we get
power active filter should compensate the instantaneous reactive power. According to the p-q theory-based approach, the shunt filter IRP p-q theory as control algorithm for shunt active filter

\[ I_a^2 + I_r^2 + I_s^2 = I^2 \]

where \( I_a, I_r, I_s \) and \( I \) denote the RMS values of \( |I_a|, |I_r|, |I_s| \) and \( |I| \). It can be seen that the second term at the right side of (8) is zero only if \( |I| \) is constant, i.e.

\[ \frac{2\pi}{0} (P_{p-} + Q_{q-}) \hat{U}(\omega t) = 0; \text{ if } |U| \text{ is constant.} \]

It means that the RMS values of the current components moduli are mutually orthogonal, i.e.

\[ I_a^2 + I_r^2 + I_s^2 = I^2, \]

only under sinusoidal voltage operation.

If the voltages waveform is not sinusoidal, then \( |U| \) is not constant and, consequently,

\[ \frac{1}{2\pi} 2\pi \int_0 P_{p-} + Q_{q-} \hat{U}(\omega t) \, d\omega = 0. \]

IRP p-q theory as control algorithm for shunt active filter

According to the p-q theory-based approach, the shunt active filter should compensate the instantaneous reactive power \( q \) and the AC component \( (p) \) of the instantaneous active power \( p \). Thus, the imposed active filter space vector current is given by

\[ I_F = I_{Fd} + jI_{Fq} = \frac{2}{3} \frac{1}{u_{d}^2 + u_{q}^2} U \left( p - jq \right), \]

and the current absorbed from the network is

\[ I_L - I_F = \frac{2}{3} \frac{u}{u_{d}^2 + u_{q}^2} P. \]

This is just the active current component in accordance with Akagi’s theory. But, in accordance with the Fryze’s, Shepherd’s and Zakikhani’s and many others’ authors’ opinions, the active current must have the same shape as the voltage [5, 7, 9, 10, 11]. It means that the square of voltage space vector modulus must be constant.

The active current under nonsinusoidal voltage conditions for active filtering reasons

In order to obtain an active current whose waveform has the same shape as the supply voltage, according to Fryze’s definition, the denominator in (4) must be constant, i.e.

\[ I_a = \frac{2}{3} \frac{P}{K^2} (u_{d} + j u_{q}). \]

But, the question arises: What is the value of this constant \( K \)? To find the answer, let us suppose a three phase nonsinusoidal voltage system which supplies a symmetrical purely resistive load \( R \) per phase. In this case, the current is purely active and thus

\[ i = \frac{u}{R} = G u. \]

In this case, the expression (2) becomes

\[ s = \frac{3}{2} u \cdot I_a^* = \frac{3}{2} G u \cdot u^* = \frac{3}{2} G |u|^2 = p. \]

Obviously, the imaginary part of \( s \) is zero. The active power is obtained as the average value of the real part \( p \), i.e.

\[ P = \frac{1}{T} \int_{-T}^{0} P_{p-} \, dt = \frac{3}{2} G \left( \frac{1}{T} \int_{-T}^{0} |u|^2 \, dt = \frac{3}{2} G U^2, \right. \]

where

\[ U^2 = \frac{1}{T} \int_{-T}^{0} |u|^2 \, dt. \]

On the other hand, starting from expression (15), the following expressions can be obtained:

\[ |I_a|^2 = G^2 |u|^2; \quad \frac{1}{T} \int_{-T}^{0} |u|^2 \, dt = G^2 U^2. \]

\[ P = \frac{3}{2} U I_a, \]

where \( I_a \) is the RMS value of the active current modulus.

Then, based on expression (14),

\[ |I_a|^2 = \frac{4}{9} \frac{P^2}{K^4 |u|^2}. \]

Accordingly, by calculating the square of \( I_a \), i.e.

\[ I_a^2 = \frac{4}{9} \frac{P^2}{K^4} U^2; \]

the active power can be expressed as follows

\[ P = \frac{3}{2} \frac{K^2}{U} I_a. \]

Then, from (20) and (23), the constant \( K \) is expressed:

\[ K = U \left( \frac{1}{T} \int_{-T}^{0} |u| \, dt \right). \]

Therefore, the correct definition of \( i \) under nonsinusoidal voltage conditions according is

\[ i_a = \frac{2}{3} \frac{P}{U^2} |U| u. \]

Certainly, expression (25) can be the general definition of the active current space vector in the p-q theory because, under sinusoidal voltage conditions, the voltage space vector modulus is constant and, therefore,

\[ U^2 = \frac{1}{T} \int_{-T}^{0} |u|^2 \, dt = |u|^2. \]

It can be seen that the definition of \( i_a \) in the p-q theory is similar to the definition proposed by Peng in [17]. Moreover, in the time domain, it is the same as the active current defined by Fryze [13]. Thus, if the phase voltage of a three phase symmetrical and balanced system is

\[ u_A = \sqrt{2} \sum_{k=1}^{N} U_k \sin k \omega t; k = 6n \pm 1, \]
Then, the voltage phasor components are

\[ u_d = u_A, \]
\[ u_q = \pm \sqrt{2} \sum_{k=1}^{N} U_k \cos(k \alpha), \]

and the square of voltage space vector modulus is

\[ |\mathbf{u}|^2 = 2 \sum_{k=1}^{N} U_k^2 \pm \sqrt{2} \sum_{k=1}^{N} U_j U_k \cos[(k \mp j) \alpha]. \]

Since

\[ \frac{1}{T} \int_{T}^{T} \cos(k \mp j) \omega t \, dt = 0; \forall k, j = 5, 7, 11, \ldots \]

the square of the RMS value of \(|\mathbf{u}|\) becomes

\[ U^2 = 2 \sum_{k=1}^{N} U_k^2. \]

For active filtering applications, our opinion is that the main concern should be directed to current compensation and not to powers compensation. Thus, when total compensation is expected, the reference current requires only the load current and its active component, in accordance with expression (1).

Case studies
Two case studies have been analyzed to serve as models to active current calculation.

A. Three phase system with nonsinusoidal voltages and symmetrical resistive load

A three phase balanced resistive load with \( R = 2\Omega \) is supplied by a three-phase distorted voltage system containing the 1st and 5th harmonics [10, 18], as follows:

\[ u_A = \sqrt{2}(100 \sin \omega t + 50 \sin 5\omega t); \]
\[ u_B = \sqrt{2}(100 \sin(\omega t - 2\pi/3) + 50 \sin(5\omega t - 2\pi/3)); \]
\[ u_C = \sqrt{2}(100 \sin(\omega t + 2\pi/3) + 50 \sin(5\omega t + 2\pi/3)). \]

Obviously, the nonsinusoidal phase voltage and supply current are in-phase (Fig.1), but the active current defined by (4) is substantially different in shape compared to the phase voltage even for a resistive load (Fig.2).

This situation occurs always under nonsinusoidal voltage conditions. Indeed, the square of voltage vector modulus \(|\mathbf{u}|^2\) is time-dependent, i.e.

\[ |\mathbf{u}|^2 = 5000(5 - 4 \cos 6\omega t). \]

This result in such a simple case study allows us to conclude that the active current component defined by (4) is not useful for reference current calculation in active filtering applications, if the voltage is strongly distorted. Indeed, if the compensation is achieved by a parallel active filter and its reference current is distorted related to the supply voltage, the RMS value of the supply current is higher than the initial load current even if this new current provides the necessary active power, removes the AC component of the instantaneous active power and has the same phase with the voltage. For example, in this case study, the RMS initial load current is exceeded by about 30% after compensation (Fig.3). Consequently, it is not a better solution. If the active current is calculated by (25), its shape is identical with the load current shape (Fig.1 and Fig.3).

In [18], professor Czarnecki gives a general example of the three phase symmetrical system with nonsinusoidal voltage (1st and 5th harmonics). He shows that the reference active filter current calculated in accordance with expression (12) is wrong. In fact, he supposed that the voltages on the phases \( R \) and \( S \) are:

\[ u_R = \sqrt{2} \left( U_1 \cos \omega t + U_5 \cos 5\omega t \right); \]
\[ u_S = \sqrt{2} \left[ U_1 \cos(\omega t - 2\pi/3) + U_5 \cos(5\omega t - 2\pi/3) \right]. \]

In accordance with Clarke transformation, he obtained the following expressions [18]:

\[ i_L = \sqrt{3} \left( U_1 \cos \omega t + U_5 \cos 5\omega t \right); \]
\[ i_L = \sqrt{3} \left[ U_1 \sin \omega t - U_5 \sin 5\omega t \right]. \]
Obviously, the active filter current vector calculated by (40) is wrong, because the load current is just active current and it must not be filtered.

Now, in accordance with our proposal, we can calculate:
- the square of the RMS value of |\underline{u}|, by (18),

\begin{equation}
U^2 = 3(U_1^2 + U_2^2);
\end{equation}

- the active power, by (17),

\begin{equation}
P = 3G(U_1^2 + U_2^2);
\end{equation}

- the active current space vector, by (25),

\begin{equation}
\underline{i}_a = \sqrt{3G} \begin{bmatrix} U_1 \cos \omega t + U_3 \cos 5 \omega t \\ U_1 \sin \omega t - U_2 \sin 5 \omega t \end{bmatrix};
\end{equation}

- the active filter current space vector, by (1), \( i_F = 0 \), because the active current calculated by proposed expression (25) is the true active current and it is just the load current.

B. Three phase full controlled rectifier-based DC motor drive system

Let us consider the DC motor supplied by a D/Y transformer via a full controlled bridge rectifier. The active filtration is made in the transformer secondary where the voltage is distorted by the rectifier commutation.

The transformer secondary phase current has the well-known waveform that outlines the effect of the limited value of the filtering inductance (Fig. 4). The current and voltage contain significant harmonics and the total harmonic distortion factor (THD) value is of 31% for the load current and of 10.38% for the supply voltage.

In fact, the Akagi’s active current has a supplementary distortion due to the commutation between phases (Fig. 5). Thus, as it can be seen in Fig. 7, the harmonics of order such as 7th, 13th, 19th, 23rd and 29th are higher than the similar ones in the proposed active current (Fig. 8).

In accordance with its definition, the complex apparent power has an important advantage, as the average value of its real part (p) is the active power P under both sinusoidal and nonsinusoidal voltage conditions. Nevertheless, a mathematical inconsequence is evident under nonsinusoidal voltage conditions. Thus, starting from complex apparent power definition and applying basic vector operations, the current vector is expressed as a sum of many terms. It is physically necessary that the term which contains the active power to denote the active component of the current. But, it does not happen, as, unfortunately, the current given by (4) is not the active current.
current under nonsinusoidal voltage conditions. Now, starting from correct active current defined by (25), we can suppose that there is an instantaneous apparent complex power

\[ s_1 = P_I + P_{1a} + j(Q_I + q_{1a}) \].

It should be natural to attach a current having the same form as (25) to each term in the right side of expression (44), i.e.

\[ i_a = 2/3 \cdot P_I / U^2 \cdot u; \]

\[ i_1 = -j \cdot 2/3 \cdot P_I / U^2 \cdot u; \]

\[ i_s = 2/3 \cdot (P_{1a} - jQ_{1a}) / U^2 \cdot u. \]

The sum of these components must be the total current,

\[ i = i_a + i_1 + i_s. \]

After replacing (45) into (46) and separating the real and imaginary parts, the following expression of new instantaneous apparent complex power is obtained,

\[ s_1 = 3/2 \cdot U^2 / |u|^2 \cdot u \cdot i \cdot i^* = U^2 / |u|^2 \cdot s. \]

If (47) is a correct definition, it must preserve the active power, i.e.

\[ \int_{T}^{T} i_p dt = U^2 \int_{T}^{T} p / |u|^2 dt. \]

Obviously, as the square of voltage space vector modulus is not always constant, condition (48) may not be accomplished.

Conclusions

The subject of the paper is a discussion on the active current extraction in three-phase three-wire systems based on the so-called "p-q theory". For total compensation reasons (harmonics and reactive power compensation), the active current of the load is required. Instead of Akagi’s active current already used in the active filter control, we have proposed an expression which can be used under nonsinusoidal voltage conditions.

Undoubtedly, the proposed active current based on the active power and on the voltage vector is useful in the calculation of the reference current for active power filters.

Thus, when total compensation is expected, the reference current requires only the load current and its active component.

The analysis of some case studies has shown that the active current which was used by Akagi is useful for compensation reasons only if the supply voltages are sinusoidal.

A modified definition of the components of the current is proposed for the operation under nonsinusoidal voltage conditions. This proposal makes possible to apply the p-q theory for active filter current calculation, under nonsinusoidal voltage.

However, under real distortion voltage conditions, when the total voltage distortion factor is less than 10%, the Akagi’s active current can be used because the difference between this current and the true active current is very small. Undoubtedly, this current is not the true active current, but, after filtering, the line current is a little distorted, as is the voltage.

The average value of the real part of Akagi’s instantaneous apparent complex power represents the active power under both sinusoidal and nonsinusoidal voltage conditions. It is a very important advantage. At the same time, there is an evident mathematical inconvenience, under nonsinusoidal voltage conditions.

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